Inferring basic parameters of the geodynamo from sequences of polarity reversals

M Fischer, G Gerbeth, A Giesecke and F Stefani

Forschungszentrum Dresden-Rossendorf, P.O. Box 510119, D-01314 Dresden, Germany

E-mail: `F.Stefani@fzd.de`

Abstract. The asymmetric time dependence and various statistical properties of polarity reversals of the Earth's magnetic field are utilized to infer some of the most essential parameters of the geodynamo, among them the effective (turbulent) magnetic diffusivity, the degree of supercriticality, and the relative strength of the periodic forcing which is believed to result from the Milankovic cycle of the Earth's orbit eccentricity. A time-stepped spherically symmetric α^2-dynamo model is used as the kernel of an inverse problem solver in form of a downhill simplex method which converges to solutions that yield a stunning correspondence with paleomagnetic data.

1. Introduction

The hydromagnetic dynamo in the Earth's outer core converts gravitational and thermal energy into magnetic energy [1]. One of the most impressive feature of the geomagnetic field is the irregular occurrence of polarity reversals. Averaged over the last few million years the mean rate of reversals is approximately 4-5 per Myr, although the last reversal occurred approximately 780000 years ago. At least two, but very likely three [3, 4] superchrons have been identified as "quiet" periods of some tens of millions of years showing no reversal at all.

Knowledge on ancient magnetic field data is mainly obtained from paleomagnetic measurements of permanent magnetization from (frozen) lava and sedimentary rocks. However, appropriate paleomagnetic sites are rare, unevenly distributed across the Earth's surface and, furthermore, actual dating methods allow only a rather rough time-resolution. Hence only few reversal characteristics can be evaluated as robust [2]. One of the commonly accepted features of reversals is their pronounced temporal asymmetry with the initial decay of the dipole being much slower than the subsequent recreation of the dipole with opposite polarity [5].

Recent numerical simulations have been successful in reproducing not only the dominance of the axial dipole and the spectrum of the geomagnetic field, but also the irregular occurrence of polarity reversals [6, 7, 8]. Polarity reversals were also observed in one [9] of the recent liquid sodium dynamo experiments which have flourished during the last decade [10].

It is important to note, however, that neither in simulations nor in experiments it is possible to accommodate all dimensionless parameters of the geodynamo [11, 12], and many of them are not even well known [13].

A complementary way to acquire knowledge about the geodynamo is to use available magnetic field data for constraining the source of dynamo action. Most famous among those attempts is the frozen-flux approximation [14, 15] which allows to infer the tangential flow at the core-mantle-boundary from secular variation of the geomagnetic field. Going beyond this frozen-flux approximation and trying to infer properties of the geodynamo in the depth of the Earth's core has been less successful so far. Such trials to "look inside the dynamo" by utilizing spectral data worked nicely for simplified dynamo models [16, 17, 18] but are hardly applicable for real world dynamos.

With this sobering experience in mind, in the present paper we undertake a rather uncommon attempt to constrain some of the most significant parameters of the geodynamo by various characteristics of paleomagnetic reversal records. Most prominent among those characteristics is the above mentioned temporal asymmetry of reversals. Two further characteristics reflect some sort of ordering in the otherwise irregular reversal sequence. The first one is the clustering property of reversals which was discovered only recently [19, 20]. Clustering of reversals is manifested in an enhanced probability of a consecutive reversal shortly after a first reversal has occurred, which results in a deviation from the Poisson distribution that would hold for uncorrelated

events. The second one is the appearance of a ∼100 kyr periodicity in the distribution of the residence times (RTD) between reversals which is believed to result from the Milankovic cycle of the Earth's orbit eccentricity [21, 22].

Based on these three input features we will examine in this paper a simplifying α^2-model of the geodynamo for which we estimate the degree of supercriticality, the noise level, the relative strength of the periodic forcing, and the effective (turbulent) magnetic diffusivity of the Earth's outer core. Actually, a few dependencies on individual parameters were already published in preceeding papers. In [23] the dependence of typical time scales on the supercriticality of the dynamo was studied, in [20] some dependencies of the clustering property on the supercriticality and the noise level were shown, and in [24] the influence of the diffusion time scale on the RTD between reversals was touched upon. What is new in the present paper is that we take all three reversal features together and try to infer from them some essential parameters of the geodynamo.

We will start with a presentation of the forward dynamo problem for which we will use a rather simple, spherically symmetric mean-field dynamo of the α^2 type. This simple model had turned out to be quite helpful for understanding the basic principle of the reversal process as a noise-induced relaxation-oscillation in the vicinity of an exceptional point of the spectrum of the non-selfadjoint dynamo operator [23, 25, 26]. This exceptional point, at which two real eigenvalues coalesce and continue as a complex conjugated pair of eigenvalues, is associated with a nearby local maximum of the growth rate situated at a slightly lower magnetic Reynolds number. It is the negative slope of the growth rate curve between this local maximum and the exceptional point that makes stationary dynamos vulnerable to noise. Then, the instantaneous eigenvalue is driven towards the exceptional point and beyond into the oscillatory branch where the sign change of the dipole polarity happens.

After having delineated the simplified mean-field dynamo model we will present the solution method for the inverse problem and the main results.

The paper concludes with a summary and a speculation on the possible consequences of our findings for the general understanding and the numerical simulations of the geodynamo.

2. The forward problem

Before tackling the inverse problem by evaluating the reversal characteristics of many different solutions of the time evolution equation for the magnetic field we have to delineate in the present section the forward problem, although this has been described already in [24].

The governing equation for the mean magnetic field **B** is the induction equation without any mean flows (**v** = 0) under the influence of a helical turbulence parameter

α [31]:

$$\frac{\partial \mathbf{B}}{\partial \tau} = \nabla \times (\alpha \mathbf{B}) + \frac{1}{\mu_0 \sigma} \nabla^2 \mathbf{B} \ . \tag{1}$$

This equation results from pre-Maxwell's equations when the source of the magnetic field generation is supposed not to be a large scale velocity $\mathbf{v}$ but some turbulence which comprises helical parts. Since the magnetic field is divergence-free we can decompose it into a poloidal and a toroidal parts according to

$$\mathbf{B} = -\nabla \times (\mathbf{r} \times \nabla S) - \mathbf{r} \times \nabla T \ . \tag{2}$$

In spherical geometry, the two defining scalars S and T are easily expanded in spherical harmonics of degree l and order m. In the following we will assume α to be spherically symmetric, being well aware of the fact that this grave simplification does not apply to the Earth's outer core. The great advantage of this simplification is that the induction equation decouples into pairs of partial differential equations for each degree l and order m,

$$\frac{\partial s_l}{\partial \tau} = \frac{1}{r}\frac{\partial^2}{\partial r^2}(rs_l) - \frac{l(l+1)}{r^2}s_l + \alpha(r,\tau)t_l \ , \tag{3}$$

$$\frac{\partial t_l}{\partial \tau} = \frac{1}{r}\frac{\partial}{\partial r}\left[\frac{\partial}{\partial r}(rt_l) - \alpha(r,\tau)\frac{\partial}{\partial r}(rs_l)\right] - \frac{l(l+1)}{r^2}[t_l - \alpha(r,\tau)s_l] \ . \tag{4}$$

where we have already used dimensionless parameters in which the radius r is measured in units of the radius of the Earth's outer core, R, the time τ in units of the diffusion time $T_d := \mu_0 \sigma R^2$, and α in units of $(\mu_0 \sigma R)^{-1}$. The boundary conditions are: $\partial s_l/\partial r|_{r=1} + (l+1)s_l(1) = t_l(1) = 0$. In the following we will consider only the dipole field with $l = 1$.

Due to the presupposed spherical symmetry of α there is no preferred direction of the magnetic field axis, hence the order m of the spherical harmonics does not show up in the equation system (3-4). This absence of a preferred direction of the dominant dipole could be considered a significant weakness of our model. Strictly speaking, our restriction to the axial dipole (the mode with $l = 1$, $m = 0$) can only be justified if some additional symmetry breaking mechanism is supposed to work. This is simply the prize we have to pay for the great advantage of remaining in the framework of only two coupled partial differential equations (3,4) for s_1 and t_1. Certainly, a study of more realistic dynamo models would be highly desirable, but with the present computer resources it will be difficult to get the good statistical validity that is easily obtainable with the large number of reversals resulting from our simple model, let alone the solution of an inverse problem as it will be presented in this paper.

The equation system (3,4), with fixed $\alpha(r)$, would represent a so-called kinematic mean-field dynamo model. Below a critical amplitude of $\alpha(r)$, the magnetic eigenfield would decay exponentially, above this value it would grow exponentially. In reality, of course, the exponential growth of the magnetic field cannot continue indefinitely. After having grown to a certain amplitude, the magnetic field attenuates the source of its own generation (Lenz's rule). While the precise way of this attenuation is an interesting

topic in its own right, we will restrict ourselves to a very simple algebraic "quenching" of the kinematic α with the angle averaged magnetic field energy which can be expressed in terms of $s(r)$ and $t(r)$. Note again, that this averaging over the spherical angle is another simplification that is chosen in order to remain in the framework of a spherically symmetric model for which the l and m decoupling of the equation (3,4) remains valid, at least formally.

While the non-linear system of equations that results from (3,4) and the algebraic quenching already exhibits a very rich behaviour we will additionally consider the influence of noise by which $\alpha(r)$ is influenced. This noise might be considered as a shorthand for fluctuations of the flow, changing boundary conditions, and the neglected influence of higher magnetic field modes.

Summarizing the quenching and the noise effect, we model the time dependent $\alpha(r, \tau)$ in the form

$$\alpha(r, \tau) = \frac{\alpha_{kin}(r)}{1 + E\left[\frac{2s_1^2(r,\tau)}{r^2} + \frac{1}{r^2}\left(\frac{\partial(rs_1(r,\tau))}{\partial r}\right)^2 + t_1^2(r, \tau)\right]} + \xi_1(\tau) + \xi_2(\tau)r^2 + \xi_3(\tau)r^3 + \xi_4(\tau)r^4 , \tag{5}$$

In this equation the noise is considered to have a finite correlation time in which it is supposed to be constant. This is equivalent to the following temporal correlation:

$$< \xi_i(\tau)\xi_j(\tau + \tau_1) > \ = D^2(1 - |\tau_1|/T_c) \times \Theta(1 - |\tau_1|/T_c)\delta_{ij} , \tag{6}$$

where Θ is the Heaviside step function. In equations (5,6), $\alpha_{kin}(r)$ is the kinematic α profile, D is the noise intensity, E is a constant measuring the inverse mean magnetic field energy, and T_c is a correlation time of the noise.

In the following we will motivate the particular choice of the $\alpha(r)$ profile. In former papers it was shown that kinematic dynamos of oscillatory character appear only in a rather small corridor of $\alpha(r)$ profiles that are characterized by at least one sign change along the radius. While this was first shown for the spherically symmetric α^2 dynamo in [18], a quite similar result was later obtained for a more realistic model in which the latitudinal dependence of α is governed by a $\cos\theta$ dependence [27]. Interestingly, such α profiles with one sign change along r were indeed found in simulations of magnetoconvection in the Earth's outer core [28].

Based on this motivation, we choose for the kinematic α profile in equation (5) the particular Taylor expansion

$$\alpha_{kin}(r) = C \cdot (\alpha_0 + \alpha_1 r + \alpha_2 r^2 + \alpha_3 r^3 + \alpha_4 r^4) \tag{7}$$

with $\alpha_1 = \alpha_3 = 0$, $\alpha_2 = -6$ and $\alpha_4 = 5$. The first coefficient, α_0, is chosen close to 1, but with two important modifications. First, we add a small parameter δ which regulates the proximity of the kinematic dynamo to oscillatory solutions. Second, α_0 will also incorporate the periodic forcing with the dominant 95 kyr period of the Milankovic cycle, being well aware of the fact that the Milankovic cycles contain also other frequencies [40]. Hence, taking both effects together we end up with the following *ansatz* for the

time-dependence of α_0:

$$\alpha_0(\tau) = 1 + \delta + \epsilon \cos(\frac{2\pi}{T_\Omega} \cdot \tau), \tag{8}$$

where ϵ parameterizes the strength of the periodic forcing. One could ask why only assuming the first Taylor expansion coefficient as time-dependent and not the entire expression on the r.h.s. of equation (7). The reason for this is that such a homogeneous scaling would have only a weak influence on the dynamo, since it results simply in a stronger quenching of $\alpha_{kin}(r)$ ending up (approximately) at the same quenched $\alpha(r)$ profile as before. In contrast to this, a selective change of only one Taylor expansion coefficient in equation (7) changes the *shape* of the $\alpha(r)$ profile, which is much more effective for changing the reversal probability. Going back to the real geodynamo, it is also not very likely that the Milankovic cycle of the Earth's orbit eccentricity changes the dynamo source homogeneously.

The equation system (3-5), with the concretization (6-8), is time-stepped by means of a standard Adams-Bashforth method with radial grid spacing of 0.02 and time step length of 2×10^{-5}. The correlation time T_c has always been set to $0.005 \cdot T_c$ which would correspond to 1 kyr in case that the diffusion time is set to 200 kyr. The resulting time series show reversal sequences quite similar to those of the geodynamo [23, 26, 29]. For the sake of simplicity, we define a reversal as the sign change of the poloidal field component $s(1, \tau)$ at the outer radius $r = 1$. Depending on the precise parameters, we get typically some 10^4 reversals in 1 day CPU time on a normal workstation. As an important characteristic of these sequences we will determine the distribution of residence times (RTD) τ_r between two subsequent reversals.

3. The inverse problem

In this section we will try to determine some of the parameters of the presented dynamo model in such a way that the resulting reversal sequences fit the characteristics of paleomagnetic data as accurate as possible.

3.1. The parameters to be determined

Being well aware of some arbitrariness in the particular choice of free parameters in the dynamo model, we have decided to choose five parameters which we believe to represent some generic features of every dynamo model.

The first one, with a comparably clear relevance and interpretation, is the magnetic Reynolds number C (that is based, however, not on the velocity but on the helical turbulence parameter α). The importance of this parameter comes from the fact that, as a measure of the supercriticality of the dynamo, it governs the typical time scale of the reversal process. This can be understood as follows: While in the rather "quiet" regime, when the dynamo is in one polarity, the value of α is quenched approximately to the critical one, amidst a reversal (when the magnetic field energy becomes small)

$\alpha(r)$ will get close to the unquenched (kinematic) profile. This means that during this short time interval the magnetic field dynamics is dominated by the possibly very large instantaneous growth rates (and frequencies) of the kinematic $\alpha(r)$ profile. Roughly speaking, the higher C the faster the reversal process happens (see, e.g., Fig. 8 in [23]). In addition to this, C governs also the asymmetry of reversals. To understand this effect imagine a kinematic $\alpha(r)$ profile which gives an oscillatory dynamo at the critical value of C. In this case, "reversals" are nothing than parts of harmonic oscillations. With increasing C, i.e. with increasing supercriticality, these oscillations become more and more anharmonic (relaxation oscillations) [26, 25]. Very often, then, there is another transition value of C beyond which the (noise-free) oscillatory dynamo becomes stationary again. However, even in this case reversals can be triggered by noise, and the asymmetry will still be governed by C.

The second parameter to be determined is the noise level, represented by D in equation (6). Roughly speaking, the larger D, the more frequent the system will leave the stable regime and undergo a reversal process. A quantitative interpretation of D is non-trivial, since for our case of finite correlation times the relevant quantity is always $D/\sqrt{T_c}$, so any interpretation of D is only sensible in combination with the correlation time of the noise.

The interpretation of the third parameter δ in equation (8) is perhaps the most intricate one. Formally, δ describes a constant shift of the kinematic α profile. In the purely kinematic regime, the proximity of the dynamo to an oscillatory solution is very sensitive on this parameter. In the highly supercritical regime, this sensitivity is reduced but there is still some influence of δ on the probability that a reversal occurs.

The fourth and the fifth parameter refer to the interplay between magnetic diffusion and the periodic forcing due to Milankovic cycle of the Earth's orbit eccentricity. This problem was already addressed in [24] where we had found that the appearance of several clear-cut maxima in the RTD at multiples of 95 kyr is hardly compatible with a diffusion time $T_d := \mu_0 \sigma R^2 = 227$ kyr. The latter would follow from recent estimates [30] of the molecular conductivity ($\sigma = 4.71 \times 10^5 \ (\Omega \, \text{m})^{-1}$) of the Fe-Ni-Si alloy at the tremendous pressure of the Earth's core. Such a large diffusion time would simply "smear out" the various maxima in the RTD which are believed to result from a stochastic resonance effect. The interesting point is now that this molecular value of the conductivity could possibly be reduced by the turbulent motion of the fluid. This so-called β effect has been estimated theoretically in various limiting cases [31]. There have also been claims [32] on the measurement of a few percent β effect in a turbulent liquid sodium flow with magnetic Reynolds numbers Rm up to 8, and some indications of it have been found in the Perm dynamo experiment [33] and the Madison dynamo experiment [34], but a clear experimental demonstration of it is still elusive. For the geodynamo with its large magnetic Reynolds number and high turbulence level, a reduction of σ by a factor 2 or so is not out of range. This is the reason why we will treat the diffusion time of the dynamo model as the fourth free parameter to be determined.

At the same time we will keep the period of the forcing fixed to 95 kyr (although

the actual forcing contains much more frequencies [40]). As indicated already in [24] we expect that the solution of the inverse problem will give us a diffusion time comparable or slightly smaller than 95 kyr, since otherwise the appearance of several maxima in the RTD is hardly explainable.

It is quite natural to consider the strength of this periodic forcing, ϵ in equation (8), as the fifth free parameter.

3.2. The functionals to be minimized and the inversion method

Since our dynamo model is quite simple, it cannot be expected to reproduce all possible reversal features. In particular, spatial distributions of the field during reversals, e.g. preferred paths of the virtual geomagnetic pole (VGP), cannot be addressed. However, we believe that typical temporal features of reversals, including the shape of individual reversals, their clustering properties and the distribution of inter-reversal times, should still be reproducible by the model. These three generic characteristics seem to us only dependent on the general reversal mechanism and on some basic parameters of the geodynamo, five of which were discussed above.

The first functional to be minimized results from the temporal shape of individual reversals which we have adopted from figure 4 of [5]. Actually, the curve "Real" in our figure 1 shows the time evolution of the virtual axial dipole moment (VADM) averaged over the last five reversals, between -53 kyr and 17 kyr. The first functional F_{shape} reflects the deviation of the simulated curves from this "real" curve:

$$F_{shape} = \sum_{i=-53}^{17} (\text{VADM}_{real}(i\Delta t) - \text{VADM}_{num}(i\Delta t))^2 \qquad (9)$$

Here, the subscript *real* refers to the averaged paleomagnetic measurement, and *num* to the numerically obtained ones. Note that for the latter we have taken an average over 100 reversal events, since we thought that an average over only five reversals would result in artificial variations with no physical significance. Δt has been set to 1 kyr, in accordance with the resolution in [5].

With the second functional we intend to accommodate the clustering property of reversal events which had been identified by Carbone et al. in 2005 [19], and analyzed in more detail in a follow-up paper by Sorriso-Valvo et al. [20]. The authors had defined a quantity which is able to detect clusters or voids even for Poisson processes with time dependent rate parameters. Consider the ith event in the sequence of reversals, and denote the shorter of the preceding or following inter-reversal intervals (or residence times) by δt, i.e.

$$\delta t = \min\{t_{i+1} - t_i; t_i - t_{i-1}). \qquad (10)$$

Then define the quantity $h(\delta t, \Delta t)$ by

$$h(\delta t, \Delta t) = 2\delta t/(2\delta t + \Delta t) \qquad (11)$$

wherein

$$\Delta t = t_{i+2} - t_{i+1} \ \ \text{if} \ \ \delta t = t_{i+1} - t_i \quad , \qquad (12)$$

$$\Delta t = t_{i-1} - t_{i-2} \ \text{ if } \ \delta t = t_i - t_{i-1} \ . \tag{13}$$

One can easily imagine that in the case of clustering it is rather likely that close to the short time interval δt there will be another short time interval Δt, hence the denominator in equation (10) will acquire a small value and the quantity $h(\delta t, \Delta t)$ will be comparably large. Evidently, h takes on values between 0 and 1, and we can either consider the probability distribution $p(h)$, or the so-called "surviving function" $P(h > H)$ that h is larger than a certain value H. For a Poisson process, with time dependent rate parameters, it was shown that the latter probability is $P_{Poisson}(h > H) = 1 - H$ [35]. The actual curve that results from paleomagnetic measurements was shown in figure 2 [19], and is also given in figure 2 of the present paper. Based on this, the second functional to be considered is

$$F_{cluster} = \sum_{i=1}^{1000} (P(h > H_i)_{real} - P(h > H_i)_{num})^2 \ . \tag{14}$$

with $H_i = i \cdot 0.001$.

While the asymmetry of reversals is rather well understood in terms of the field dynamics in the vicinity of an exceptional point of the spectrum, the physical reason of the clustering property is less obvious. In [26, 20] we had speculated that the appearance of clusters, which can be visualized as a "devil's staircase" of reversal events [36], is a typical sign of "punctuated equilibrium" [37] and self-organized criticality [38]. In this respect it is also interesting to note the tendency of the clustering property to increase with the degree of super-criticality [20].

The third functional refers to the distribution function of inter-reversal times (or residence times). The appearance of certain periods in the reversal sequence had been controversially discussed for a long time [39, 40, 41]. A breakthrough was achieved by Consolini and DeMichelis who analyzed not the Fourier spectrum but the probability distribution of inter-reversal times [21]. In this distribution they observed a clear sequence of several maxima at multiples of approximately 95 kyr. The physical reason for these maxima was identified as a stochastic resonance with the Milankovic cycle of the Earth's orbit eccentricity [22]. The data from [21] are shown again in figure 3, and the corresponding functional is

$$F_{rtd} = \sum_{i=1}^{90} (p(\tau_i)_{real} - p(\tau_i)_{num})^2 \tag{15}$$

with $\tau_i = i \cdot 1$ kyr.

Obviously, having defined the three functionals, there still remains an ambiguity in the choice of the relative weights for them. We will test some reasonable choices of the a-priori errors σ_{shape}, $\sigma_{cluster}$, and σ_{rtd} in the total functional

$$F_{total} = \sigma_{shape}^{-2} F_{shape} + \sigma_{cluster}^{-2} F_{cluster} + \sigma_{rtd}^{-2} F_{rtd} \tag{16}$$

and check afterwards the correspondence of the resulting curves with the paleomagnetic ones.

The very inversion is carried out by using a standard downhill simplex method taken from "Numerical Recipes" [42]. Since we have 5 parameters to be determined, we

Table 1. Parameters of the three versions resulting from the downhill simplex inversion.

	C	δ	C_{crit}	C/C_{crit}	D	T_d	ϵ	#/Myr
Version 1	57.0	-0.03	13.0	4.4	6.3	86.8 kyr	0.133	2.05
Version 2	96.8	-0.16	8.99	10.8	8.0	63.7 kyr	0.123	3.07
Version 3	147.8	-0.18	8.73	16.9	7.8	55.1 kyr	0.094	2.91

use a simplex with 6 points. For each of those points in parameter space we solve the forward problem for 20000 diffusion times which typically gives a few thousand reversals. From these reversals the functional (16) is computed, and based on this evaluation the usual steps of the downhill simplex method are performed, until an appropriate stopping criterion is reached. While an individual run takes a few hours on a normal PC, the solution of the inverse problem takes about one week.

3.3. Results

In the following we will present three solutions which result from choosing different weights of the three individuals functionals in (16).

Table 1 shows the obtained parameters of the three versions. The second raw gives the parameter C, the third raw the shift parameter δ. In the fourth raw the critical value of the dynamo for the $\alpha(r)$-profile with $\alpha_0 = 1 + \delta$, $\alpha_1 = \alpha_3 = 0$, $\alpha_2 = -6$ and $\alpha_4 = 5$ (see Eqs. 5-7) is shown, which differs from version to version due to the variation of δ. The fifth raw gives the degree of supercriticality, i.e. C/C_{crit}, where C_{crit} prescribes the amplitude of the critical α at which the onset of dynamo action occurs. The raws 6, 7 and 8 show the noise level D, the (effective) diffusion time T_d, and the strength of the periodic forcing ϵ, while the last raw gives the resulting number of reversals per Myr for each version.

Figures 1, 2 and 3 show the curves for the temporal dependence of the VADM, for the probability $P(h > H)$, and for the RTD between reversals, compared with the corresponding curves from the paleomagnetic data.

Obviously, version 2 gives the best correspondence with all three reversal characteristics. Compared to this "optimal" version 2, in version 1 σ_{rtd} was increased by a factor 2 which results in a deteriorated correspondence with the real RTD curve in figure 3. In version 3, σ_{shape} was increased by a factor 2 (compared to version 2), which leads to a deteriorated correspondence with the real VADM curve in figure 1. Evidently, the large value of C and the small T_d provide reversals that are faster than the really observed ones.

4. Conclusions

With version 2, we have obtained the best reproduction of the paleomagnetic input data for a 10 times supercritical dynamo, a relative strength of the periodic forcing
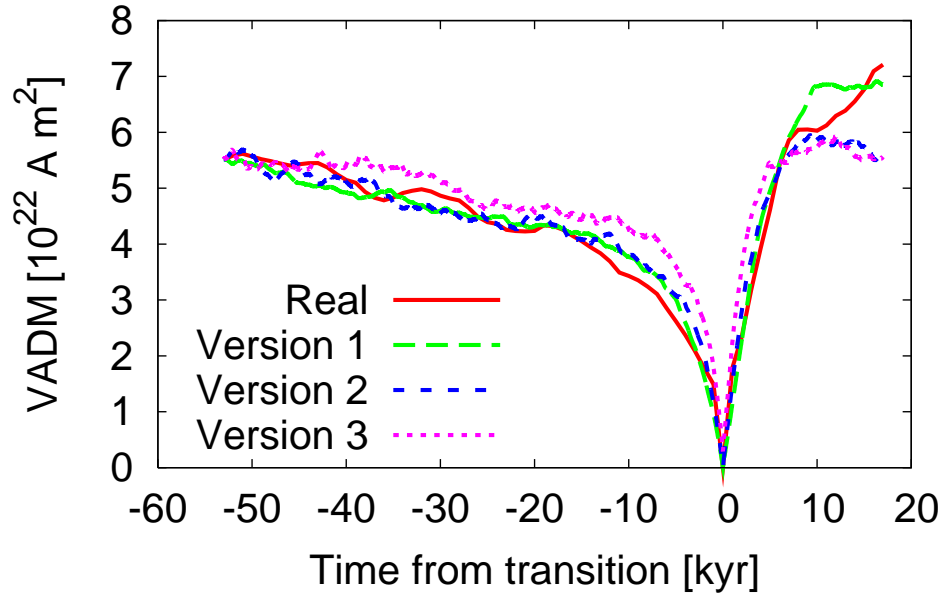
Figure 1. Temporal evolution of the virtual axial dipole moment (VADM) during reversals. Comparison of the average over the last five real reversal data (taken from [5]) with the average over 100 reversals for the three versions resulting from different solutions of the inverse problem.
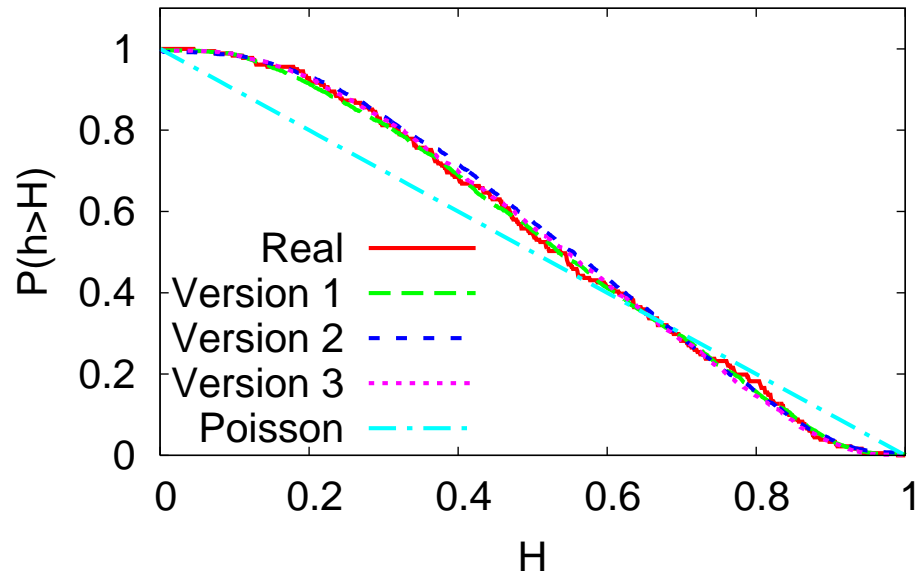

Figure 2. Surviving function $P(h > H)$ for the real data (adopted from [19, 43]) and for the three versions. The deviation from the straight line which would appear for a Poisson process indicates a significant clustering.

of some 15 per cent, and an effective magnetic diffusion time of approximately 65 kyr which is by a factor 3.5 smaller than the value that would result from the molecular
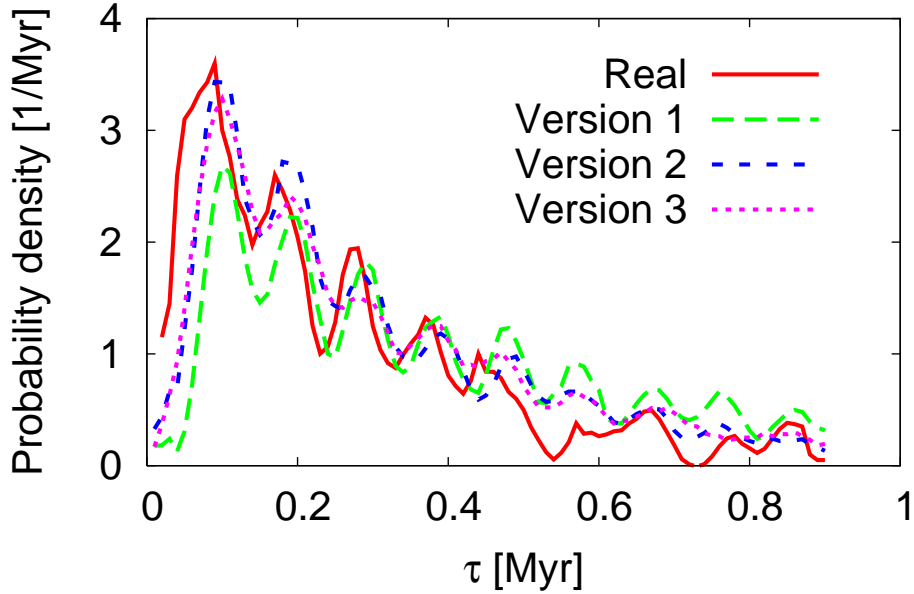
Figure 3. Residence time distribution (RTD) for the real data (taken from [21]) and for the three versions.

conductivity. The latter is perhaps the most important result of the inversion, for the following reason: The conductivity enhancing effect of turbulence is a subject of ongoing debate. Only very few estimations exists for the turbulent diffusivities within the Earth's outer core. However, their values are of extraordinary importance for the key parameters of the geodynamo (e.g. Ekman number or the magnetic Prandtl number) as well as for the estimation of turbulent transport properties and the destructive influence of turbulence on magnetic field generation through turbulent field diffusion. The molecular values of the dimensionless parameters are extremely small and cannot be realized in simulations so that usually enhanced values are applied that shall resemble the effective (i.e. turbulent) quantities. A more detailed information on the turbulent diffusivities therefore delivers important information on the significance of the parameter space accessible in global MHD simulations for the geodynamo. Furthermore, reality is more complicated, as it is very likely that a conductivity reduction due to turbulence would be anisotropic, because the small scale turbulence in a fast rotating object like the Earth is subject to preferred directions parallel to the rotation axis (and also along the dominating field component) so that the convection cells (that determine the turbulent transport of physical quantities) are oriented parallel to the rotation axis and elongated along the magnetic field. It is well known that an anisotropic conductivity could have a tremendous effect on the selection between equatorial and axial dipole solution [44, 45]. Roughly speaking, axially aligned rotating columns tend to decrease the conductivity for horizontal currents less than the conductivity for vertical currents, and this effect leads to a preference of the axial dipole compared to the equatorial dipole. This could be an extremely important point since the conclusions of many geodynamo related papers

rely on an isotropic conductivity when looking for criteria for the selection of axial or equatorial dipoles.

The stunning agreement of paleomagnetic and numerical reversal characteristics as shown in figures 1, 2 and 3 gives support to our hypothesis that reversals are indeed noise triggered relaxation oscillations in the vicinity of an exceptional point of the spectrum of the dynamo operator. In this respect it is important to note that in particular the time asymmetry and the clustering property are intrinsic and robust properties of the model that appear for very wide regions of parameter (if not for all). By solving the inverse problem we have not "produced" them, but have only fine-tuned the dynamo parameters to fit optimally the paleomagnetic data.

We have carefully tried not to over-interpret our simple model by focusing only on those parameters to be determined, and those functionals to be minimized, that refer to the temporal properties of reversal sequences, and not to any spatial features. This makes us optimistic that the results will prove robust when inversions of this kind will later be repeated using more realistic dynamo models. Given that one downhill simplex run for our simple model takes already one week, one can imagine that corresponding runs with better dynamo models will lead to significant computational costs.

Acknowledgments

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